FILM FLOW OF MAGNETIC LIQUID IN THE LAMINAR REGIME

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The hydrodynamics of the film flow of a magnetic liquid over reflux surfaces in static magnetic fields is studied.

When the ponderomotive interaction of a magnetic liquid with the magnetic field is taken into account, the Navier–Stokes equation is modified in the following manner [1-3]:

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \eta\nabla^2\mathbf{v} + M\nabla B + \rho\mathbf{g}.$$

1. In the case of the slow, steady-state, laminar draining of a magnetic film of constant thickness along a vertical reflux surface in an inhomogeneous magnetic field with a vertical induction gradient, the equation for the sole nonvanishing velocity component becomes [4]

$$\eta \frac{d^2v}{dy^2} - \rho g + M\frac{dB}{dz} = 0.$$

Assuming $M\Delta B$ = const, and assuming that the velocity at the reflux surface and the tangential stresses on the free surface vanish, we find

$$v = \frac{1}{\eta}\left(M\frac{dB}{dz} - \rho g\right) y \left(a - \frac{y}{2}\right).$$

Depending on the relation between the magnetic and gravitational forces, the film may either fall or rise. In the case $M(dB/dz) = \rho g$, the film cannot move. In the saturation state, the cutoff value of the magnetic force is governed solely by the gradient of the magnetic induction. Assuming $\rho = 2\ g/cm^3$, $g = 10\ m/sec^2$, and $M = 4\cdot 10^5$ A/m, in accordance with [5], we find a value of $5\cdot 10^{-2}$ T/m for the critical gradient of B.

2. We consider the steady-state motion of a magnetic field along a reflux surface in an axisymmetric magnetic field. The normal to this surface is collinear with **g**. By analogy with [6], we can determine the radial velocity component approximately from

$$\eta \frac{d^2v}{dz^2} + M\frac{dB}{dr} = 0,$$

which is written under the condition that the z axis of the cylindrical coordinate system coincides with the symmetry axis of the magnetic field. Integrating this equation, we find

$$v = \frac{M}{\eta}\frac{dB}{dr} z\left(\delta - \frac{z}{2}\right). \tag{1}$$

In the case $dB/dr > 0$ (a quadrupole magnetic lens [7]), the film moves away from the axis toward the periphery of the reflux surface. In the case $dB/dr < 0$ (a current-carrying conductor), the film flows away from the periphery toward the axis. From the integral continuity equation

$$2\pi r \int_0^\delta v dr = Q = \text{const}$$

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we find the variable film thickness:

$$\delta = \lambda\varphi(r), \quad (2)$$

where

$$\lambda = \sqrt[3]{\frac{3\eta Q}{2\pi}}, \qquad \varphi(r) = \left(rM\frac{dB}{dr}\right)^{-1/3}.$$

For an infinite, rectilinear conductor carrying a current I we find

$$\varphi(r) = -\sqrt[3]{\frac{4\pi^2}{\mu\mu_0(\mu-1)I^2}}\, r^{2/3}$$

for weak magnetic fields and

$$\varphi(r) = -\sqrt[3]{\frac{2\pi}{\mu_0 MI}}\, r^{1/3}$$

for strong magnetic fields. In the field of a quadrupole magnetic lens, we have

$$\varphi(r) = \sqrt[3]{\frac{1}{\mu\mu_0(\mu-1)b^2}}\, r^{-2/3}$$

or

$$\varphi(r) = \sqrt[3]{\frac{1}{\mu_0 MB}}\, r^{-1/3}.$$

3. In the quasisteady approximation [8, 9], for the case of axisymmetric motion of the magnetic film, the film thickness is found from the differential continuity equation

$$\frac{\partial h}{\partial t} + \frac{1}{2\pi r}\cdot\frac{\partial Q}{\partial r} = 0. \quad (3)$$

The volume flow rate of the liquid per unit time is found from (1) to be

$$Q = \frac{2\pi rM}{3\eta}\cdot\frac{dB}{dr}\, h^3.$$

We assume that the magnetic film moves in the field of a quadrupole magnetic lens under conditions such that the field can be assumed weak. Then we have

$$Q = mr^2h^3, \quad (4)$$

where $m = (2\pi/3\eta)\mu\mu_0(\mu-1)b^2$. Substituting (4) into (3), we find

$$\frac{\partial h}{\partial t} + \frac{3m}{2\pi} rh^2\frac{\partial h}{\partial r} + \frac{m}{\pi} h^3 = 0.$$

Following [8], we seek a particular solution of this equation in the form $h = kr^{\alpha}t^{\beta}$. It turns out that

$$h = \sqrt{\pi/2mt}. \quad (5)$$

Following an analogous procedure for strong magnetic fields, we find

$$h = \sqrt{2\pi r/5nt}, \quad (6)$$

where $n = (2\pi/3\eta)\mu_0 bM$. Solutions (5) and (6) hold if r and t are sufficiently large. Solution (6) differs by only a numerical coefficient from the familiar Jeffreys solution [8] for the thickness of a heavy film draining along a vertical plate. The reason for this agreement is that the magnetization is constant in a strong magnetic field, and the liquid moves under the influence of a constant force.

NOTATION

g is the gravitational field;
ρ is the density of liquid;
η is the dynamic viscosity;
M is the magnetization of liquid;
B is the magnetic induction;
μ is the magnetic permeability of liquid;
μ_0 is the permeability of free space;
a is the constant film thickness;
r is the radial coordinate of the cylindrical coordinate system;
δ is the steady-state variable thickness of film;
h is the unsteady variable thickness of film;
Q is the volume flow rate of liquid (per unit time);
I is the current;
b is the constant gradient of magnetic field in quadrupole lens.

LITERATURE CITED

1. J. L. Neuringer and R. E. Rosensweig, Phys. Fluids, 7, 1927 (1964).
2. V. G. Bashtovoi and B. M. Berkovskii, Magnitn. Gidrodinam., No. 3 (1973).
3. M. I. Shliomis, Usp. Fiz. Nauk, 112, 427 (1974).
4. V. G. Levich, Physicochemical Hydrodynamics, Prentice-Hall, Englewood Cliffs, New Jersey (1962).
5. R. Kaiser and G. J. Miskolczy, J. Appl. Phys. 41, 1064 (1970).
6. K. Geizli and A. Charwat, Heat and Mass Transfer [Russian translation], Vol. 10, Nauka i Tekhnika Minsk (1968), p. 401.
7. L. A. Artsimovich, S. Yu. Luk'yanov, Motion of Charged Particles in Electric and Magnetic Fields [in Russian], Nauka, Moscow (1972).
8. H. Jeffreys, Proc. Cambr. Phil. Soc., 26, 204 (1930).
9. C. Gutfinger and Tallmadge, AIChE J., 10, 774 (1964).